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Edge excitations of a two-dimensional electron solid in a magnetic field

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Abstract. The effect of the solidification transition on the edge phenomena of a two-dimensional (2D) electron system in a magnetic field is studied in the classical and quantum melting regimes. It is shown that in the pure solid system ($T \to 0$), instead of the conventional edge magnetoplasmons (EMP) which are observed in the liquid state, new magneto-Rayleigh waves (MRW) can propagate along the solid boundary and these have much lower frequencies: $\omega_{MRW} \ll \omega_{EMP}$. If at a finite temperature $T < T_c$ the solidification of the edge is not completed due to the smooth density profile, the EMP and the new boundary displacement waves can propagate in a thin electron liquid strip surrounding the 2D solid. The properties of these waves are affected by the solid boundary which may provide an alternative approach for probing the Wigner transition.

1. Introduction

In the presence of a strong magnetic field ($\boldsymbol{B}$) oriented normally, the edge phenomena of the two-dimensional (2D) electron liquid have been studied intensively because of the unique properties of such plasma oscillations, which are confined to the boundary [1–3], and of their relationship to such fundamental issues as the quantum Hall effect (QHE) [4, 5]. Conventional edge magnetoplasmons (EMP) as well as the new multiple acoustic excitations [6, 7] and the recently observed boundary displacement waves coupled with EMP [8, 9] are very important for the understanding of the edge current states of the QHE, since they relate to a transition strip where the properties of the 2D electron system change drastically.

Another fundamental issue studied in 2D electron systems is the Wigner solid. Yet the edge waves of such a 2D electron solid have not been analysed. In the present paper we show that a variety of waves, some of which are of a new kind, can propagate along the boundary of a 2D electron solid in the presence of a magnetic field.

The Wigner solid state of a 2D electron gas was first observed experimentally for electrons on the surface (SE) of superfluid helium [10]. The system is primarily in the classical melting regime, with the liquid–solid transition being characterized by the critical value of the classical plasma parameter

$$\Gamma_m = \frac{e^2\sqrt{\pi n_0}}{k_B T_m} \simeq 140.$$

Here T_m is the melting temperature, and n_0 is the mean electron density. The edge of the system is usually fixed by the external field of the guard electrodes. As shown in figure 1,

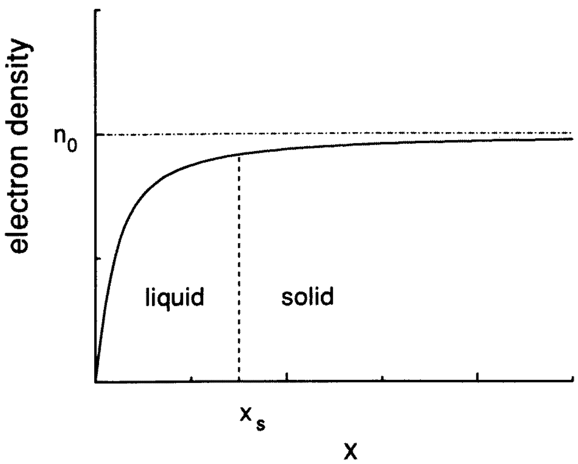

Figure 1. A sketch of the electron density profile $n(x)$ and the phase separation at the edge below the solidification transition.

in this case the density profile at the edge $n(x)$ is smoothed over a distance d ($n(x) \to n_0$ at $x \gg d$) which is of the same order of magnitude as the distance between the helium surface and the bottom electrode, H. Below the transition point, $T < T_m$, the electrons which are close enough to the edge do not satisfy the Wigner solid condition $\Gamma \geqslant \Gamma_m$ and should be in the liquid state. This means that the sharp edge of a solid, placed at $x = x_s(T)$ (here x_s is the solution of the equation $n(x) = n_m$, and $\sqrt{n_m} = k_B T \Gamma_m / (\sqrt{\pi} e^2)$), should be surrounded by a thin strip of electron liquid ($0 < x < x_s$). We will see that in this case two kinds of motion are possible due to a significant difference between the excitation frequencies. One motion is just the fluid oscillations within the surrounding strip. Another motion involves the displacements of the solid boundary.

In section 2 we study the idealized case of a 2D Wigner solid with a sharp edge which is important for the analysis of the excitation spectrum of a real system. The Wigner transition substantially changes the bulk excitation spectrum of a 2D electron system in a magnetic field. In addition to the high-frequency mode with $\omega = \omega_+(k) > \omega_c$ (here ω_c is the cyclotron frequency), a low-frequency bulk mode $\omega = \omega_-(k) \propto 1/B$ [11] appears, which crucially affects the propagation of edge waves. For instance, the conventional EMP cannot propagate along the edge of a pure Wigner solid, since their frequencies are within the bulk excitation spectrum. The only possible solution in this case is the magneto-Rayleigh wave (MRW) whose frequency has the same B-dependence, but is much lower in frequency than the EMP.

For a real density profile (the electron concentration smoothly decreases towards the edge) at a non-zero temperature the edge of the 2D Wigner solid should be surrounded by a narrow strip of liquid. In this case the boundary displacement wave (BDW) of the liquid strip (section 3) and the EMP (section 4) can propagate along the wet edge of the system. The appearance of the liquid–solid interface at the edge of the 2D electron system changes the conditions for the wave propagation on both sides of the interface, which can be used as an experimental tool to study the Wigner solid. Furthermore, the order of phase separation at the edge inverts if the electron system is going from the classical melting regime to the quantum regime, and vice versa. This effect can also be used for probing the 2D electron solid.

2. The magneto-Rayleigh waves

As is clearly seen in figure 1, the 2D electron solid has a sharp edge for temperatures just below the transition point ($T < T_m$). It is obvious that in the case of a movable boundary (electrons are confined by means of the electric field of the guard electrodes) the thin liquid strip cannot substantially affect the edge excitations of the solid. Therefore, to simplify the problem we will first neglect the effect of the thin liquid strip on the solid motion and choose the origin of the x-axis to be at the edge of the solid ($x_s \equiv 0$).

According to [11–13], the equations of motion of a Wigner solid are practically the same for the SE on helium and 2D electrons in a semiconductor system. Therefore, to avoid unimportant details of a particular system, we will generally express our results in terms of the frequencies of the basic modes of a 2D electron solid.

Any elastic wave propagating along the solid boundary should be a solution of the bulk equations of motion

$$-\omega^2 \boldsymbol{u}_{\boldsymbol{k}} + \mathsf{D}_{\boldsymbol{k}} \boldsymbol{u}_{\boldsymbol{k}} - \mathrm{i}\omega[\boldsymbol{\omega}_c \times \boldsymbol{u}_{\boldsymbol{k}}] = 0 \tag{1}$$

where the electron displacements $\boldsymbol{u}$ from the equilibrium lattice positions $\boldsymbol{R}$ are taken to be a monochromatic wave $\boldsymbol{u} = \boldsymbol{u}_{\boldsymbol{k}} \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{R} - \mathrm{i}\omega t)$; $\boldsymbol{\omega}_c$ ($\omega_c = eB/mc$; m is the electron mass) is directed along the magnetic field; $D_{\boldsymbol{k}}^{\alpha\beta}$ is the dynamical matrix which in the long-wavelength limit can be expressed in terms of the transverse ($p = t$) and longitudinal ($p = \ell$) mode frequencies ω_p:

$$D_{\boldsymbol{k}}^{\alpha\beta} = \omega_t^2(k)\delta_{\alpha\beta} + [\omega_\ell^2(k) - \omega_t^2(k)]k_\alpha k_\beta/k^2.$$

The dispersion equation which follows from equation (1) can be written in the form $(\omega^2 - \omega_-^2(k))(\omega^2 - \omega_+^2(k)) = 0$, where $\omega_\pm(k)$ are the bulk magnetoplasmons of a 2D electron solid [11]. The properties of these modes are well known. The most important one is that $\omega_-(k)$ decreases with increasing magnetic field B and in the limit of strong fields is characterized by the asymptotic behaviour $\omega_-(k) \cong \omega_\ell(k)\omega_t(k)/\omega_c$. It should be emphasized that $\omega_-(k)$ is much lower than typical EMP frequencies of a 2D electron liquid which are usually of the order of $\omega_\ell^2(k)/\omega_c$.

We will confine ourselves to the limiting case $a/2 < H \ll |k_y|^{-1}$ (here a is the lattice spacing) which corresponds to the real experimental situation for SE on helium and allows us to use boundary conditions of elasticity theory. In this case both longitudinal and transverse phonons are of acoustic nature [12, 13]: $\omega_p(k) = c_p k$, where

$$c_\ell^2 \cong (4\pi e^2 n_0 H/m)(1 - 0.09a/H)$$
$$c_t^2 \cong 0.138 e^2 \sqrt{\pi n_0}/m.$$

We search for solutions to equation (1) which are confined to the boundary, and therefore we take $\boldsymbol{k} = \{\mathrm{i}s|k_y|, k_y\}$. Equation (1) yields two possible solutions for the function $s(\omega)$:

$$s_\pm = \left\{1 - \frac{\omega^2}{2\omega_\ell^2\omega_t^2}[\omega_\ell^2 + \omega_t^2 \mp (\omega_\ell^2 - \omega_t^2)W]\right\}^{1/2} \tag{2}$$

where we have introduced the notation

$$W = \left[1 + \frac{4\omega_\ell^2\omega_t^2\omega_c^2}{\omega^2(\omega_\ell^2 - \omega_t^2)^2}\right]^{1/2}$$

where $\omega_t = c_t|k_y|$ and $\omega_\ell = c_\ell|k_y|$. To satisfy the boundary conditions, both solutions $s_\pm$ should contribute to the lattice displacements:

$$\boldsymbol{u}(x, y) = \{\boldsymbol{A}_+ \exp(-s_+|k_y|x) + \boldsymbol{A}_- \exp(-s_-|k_y|x)\} \exp(\mathrm{i}k_y y) \tag{3}$$

where we used the continuum approximation. According to equation (1), the components $A_{\pm}^{(\alpha)}$ of the vectors $\boldsymbol{A}_{\pm}$ are related as follows:

$$A_{\pm}^{(x)} = Q_{\pm}(\omega) A_{\pm}^{(y)} \qquad Q_{\pm}(\omega) = \frac{(\omega_{\ell}^2 - \omega_t^2) s_{\pm}(\omega) - \omega\omega_c \,\mathrm{sgn}\{k_y B\}}{\omega^2 - \omega_t^2 + \omega_{\ell}^2 s_{\pm}^2(\omega)}. \tag{4}$$

The necessary condition for the edge wave to be exponentially damped towards the interior of the solid is that the frequency of the wave should be lower than the bulk spectrum frequency ($\omega < \omega_-(|k_y|)$).

The relation between the frequency and k_y is found by making use of the appropriate boundary conditions. At the free boundary the stress tensor satisfies $\sigma_{str}^{\alpha y}(x \to 0) = 0$. According to 2D elasticity theory, these boundary conditions can be written as follows:

$$\frac{\partial u^{(y)}}{\partial x} + \frac{\partial u^{(x)}}{\partial y} = 0 \qquad (1 - 2\delta)\frac{\partial u^{(y)}}{\partial y} + \frac{\partial u^{(x)}}{\partial x} = 0 \tag{5}$$

where $\delta = c_t^2/c_{\ell}^2$. It is interesting to note that, if we impose the rigid boundary condition at the edge, $u^{(x)}(x \to 0) = 0$ (typical for the EMP), instead of the second boundary condition of equation (5), the final dispersion equation $s_+(\omega)Q_-(\omega) = s_-(\omega)Q_+(\omega)$ will have no low-frequency solution at all.

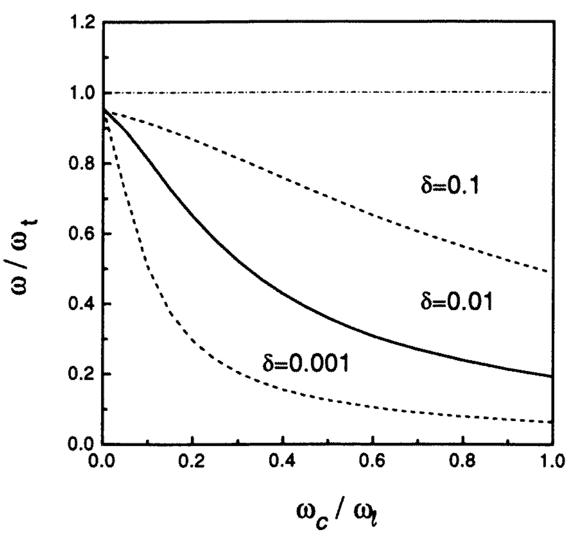

Figure 2. The MRW spectrum versus the magnetic field (in units of ω_c/ω_{ℓ}) for three values of the ratio $\delta = c_t^2/c_{\ell}^2$.

The movable boundary conditions of equation (5), which are more suitable for the system confined by means of the external field of the guard electrodes, yield the following dispersion equation:

$$(1 - 2\delta + Q_- Q_+)(s_+ - s_-) = (1 - 2\delta + s_- s_+)(Q_+ - Q_-). \tag{6}$$

In the zero-magnetic-field limit ($\omega_c = 0$) the solution to this equation is just the Rayleigh wave of a 2D solid: $\omega/\omega_t \to 0.955$ at $\delta \to 0$. For weak and intermediate magnetic fields the frequency of the edge MRW is shown in figure 2 as a function of the magnetic field. It is clearly seen that the effect of the magnetic field on the spectrum of the MRW increases (in units of ω_c/ω_{ℓ}) with decreasing δ (increasing H).

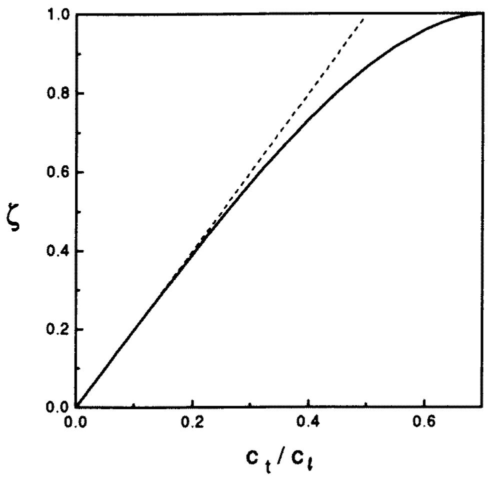

Figure 3. The solution of equation (7) (solid curve) and the asymptote $\zeta = 2\sqrt{\delta}$ (dashed line) versus the ratio c_t/c_ℓ.

In the most interesting limiting case of strong magnetic fields $\omega_c \gg \omega_\ell$ the field dependence of the MRW spectrum is of the form $\omega = \zeta\omega_\ell\omega_t/\omega_c$, where ζ is the solution of the following transformed equation:

$$\mathrm{sgn}\{k_y B\}\sqrt{\delta}(\sqrt{1+\zeta} - \sqrt{1-\zeta})[(1-\delta)^2 - \zeta^2/2] = (\sqrt{1-\zeta^2} - \delta)\zeta^2/2. \tag{7}$$

As a result of the transformation a false solution $\zeta = 1 - \delta$ was introduced which is just the root of the denominator of $Q_-(\omega)$ and therefore should be dismissed. In the limit of small $\delta \ll 1$, which is quite usual for a 2D electron solid, the real solution of equation (7) represents the MRW and can be found analytically: $\zeta \cong 2\sqrt{\delta}$. This result satisfies the above-mentioned condition $\omega < \omega_-(k)$ necessary for the displacements to be localized at the edge. Formally, at larger δ the solution deviates from this asymptote as is shown in figure 3 and vanishes at $\delta \to 1/2$, with the 2D Poisson ratio $1 - 2\delta \to 0$.

The dispersion curve of the 2D MRW can be plotted in a rather general way as shown in figure 4 by the use of the universal wave-vector parameter $k_c = \omega_c/c_\ell \propto B$. In the strong-field limit ($k_c \gg k_y$), according to figure 4, we have $\omega \propto k_y^2$. For larger k_y the effect of the magnetic field decreases and finally the wave attains the acoustic spectrum $\omega \propto k_y$.

It should be emphasized that for given $\boldsymbol{B}$ the low-frequency MRW solution of equation (7) exists only for one definite direction of wave propagation ($\mathrm{sgn}\{k_y B\} = +$). This behaviour is typical for EMP waves in a 2D electron liquid. Still the frequency of the MRW is much lower than that of conventional EMP and BDW. This means that the solid motion cannot follow the high-frequency oscillations of the EMP and BDW propagating along the liquid strip which surrounds the solid boundary. In other words, when the liquid motion is considered, the liquid–solid interface can be treated as a wall.

Regarding the damping of the MRW, we expect that it will be determined by the conductivity of the electrons in a particular system, since in the long-wavelength limit the electron motion penetrates deep into the 'bulk' of the 2D solid, and any microscopic defect at the edge should not seriously affect the wave propagation. Therefore the conditions for the observation of the MRW are the same as those for the observation of the 'bulk' transverse mode, at least in the intermediate-magnetic-field regime ($\omega_c \sim \omega_\ell$). The required

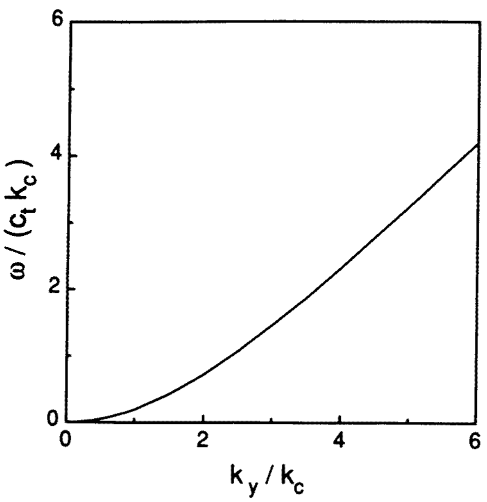

Figure 4. The dispersion curve of the MRW in terms of the field-dependent wave-vector parameter $k_c = \omega_c / c_\ell$.

smoothness of the edge (the size of the defect $L_d \ll k_y^{-1}$) and the absence of static defects at the edge are usually achieved for SE or ions bound to the surface of superfluid helium. In such systems very large mean free paths have been measured for edge wave propagation.

3. The boundary displacement waves of the wet edge

As follows from the analysis presented in the introduction, the BDW and EMP resonances do not disappear at the Wigner solid transition in the classical melting domain because of the liquid strip which wets the edge of the 2D electron solid.

Two kinds of collective excitation are possible within the liquid strip. The EMP is a density wave confined to the rigid edge of a 2D electron system; the current component in the x-direction is assumed to be zero at the edge. The BDW is analogous (not equivalent!) to the capillary wave of an ordinary liquid. In general these motions are coupled in a sort of in- and out-of-phase oscillation of the edge charges [8, 9]. The coupling is strong if the frequencies of these modes are close enough, which is the case for a sharp density profile at the edge of an unscreened Coulomb system. For a rather smooth density profile, EMP and BDW have substantially different frequencies [9]. Therefore an independent treatment of these modes in the strip can be considered as a reasonable approximation. Since the EMP in a strip geometry were studied theoretically [3] we will confine ourselves primarily to properties of the BDW.

First we will use a model approximation in which a liquid strip of a constant density n_0 and of a width d is bound to a stiff wall of the solid phase. For a usual liquid bound to a wall we could expect a considerable reduction of the excitation frequency in the long-wavelength limit ($|k_y| d \ll 1$) described by the factor $\sqrt{\tanh(|k_y| d)}$. But this is not the case for BDW in the strong-magnetic-field limit.

In the frequency range beyond the typical EMP frequencies, the 2D electron system can be treated as an incompressible electron liquid: $\mathrm{div}(\boldsymbol{V}) = 0$ with the velocity field $\boldsymbol{V} = \nabla\varphi$, where φ is the hydrodynamic potential of the liquid. It should be noted that there are no modes in the bulk of the incompressible liquid. We use the Euler equation

$$\mathrm{i}\omega V^{(y)} = \frac{1}{mn_0}\frac{\partial \Psi}{\partial y} + \mathrm{sgn}\{B\}\omega_c V^{(x)} \tag{8}$$

where $\Psi = p + en_0\Phi_\xi$; p is the liquid pressure; Φ_ξ is the perturbation of the electrical potential caused by the boundary displacements $\xi(y)$. The continuity condition for the pressure at the edge gives the effective boundary condition $\Psi(x \to 0) = en_0\Phi_\xi(x \to 0) \equiv \alpha e^2 n_0^2 \xi$, where α is a geometrical factor of the order of one [9].

The boundary conditions for the velocity field are quite usual: $V^{(x)}(x \to 0) = -\mathrm{i}\omega\xi$, and $V^{(x)}(x \to d) = 0$. The first one is the free boundary condition, while the second one is the condition at a 'wall'. In this case the dispersion equation which follows from equation (8) can be written as

$$\omega[\omega + \mathrm{sgn}\{Bk_y\}\omega_c \tanh(|k_y|d)] = \omega_0^2(k_y)\tanh(|k_y|d) \tag{9}$$

where $\omega_0^2(k_y) = \alpha(e^2/m)n_0|k_y|$. For $k_y > 0$ equation (10) has only one physically acceptable solution:

$$\omega = \frac{1}{2}\left[\sqrt{\omega_c^2 + 4\omega_0^2(k_y)\coth(|k_y|d)} - \mathrm{sgn}\{Bk_y\}\omega_c\right]\tanh(|k_y|d) \tag{10}$$

which for $\mathrm{sgn}\{Bk_y\} = +$ represents the low-frequency BDW. In the zero-magnetic-field limit the frequency of this wave has the typical $k_y d$-dependence: $\omega \simeq \omega_0(k_y)\sqrt{\tanh(|k_y|d)}$, while in the strong-field limit we still have $\omega \simeq \omega_0^2/\omega_c$. Yet to achieve this limiting behaviour, much stronger fields are necessary: $\omega_c \gg \omega_0(k_y)\sqrt{\coth(|k_y|d)}$.

In a real system the liquid strip is not of constant density. Therefore the previous result (that the BDW frequency is independent of d for strong fields) cannot be applied directly to the case of a real liquid density profile. Still there are some hints which show that qualitatively this behaviour of the BDW might remain if $\xi \ll d$: for a narrow strip $p \approx 0$ and the Euler equation becomes independent of n_0, while for an arbitrary density profile $\Phi_\xi(0)$ still has the form $\alpha n_0 e\xi$, independently of the phase state of the entire electron system.

4. The EMP affected by the Wigner solid boundary

For the classical melting regime the phase order at the edge is shown in figure 1. It should be noted that usually the density perturbations of conventional EMP are spread over the whole density profile transition width $0 < x \leqslant d$. The appearance of the solid boundary within this width ($x_s < d$) does not allow the density perturbations to appear at $x > x_s$ due to the properties of the MRW discussed above. Indeed, we must impose the 'wall' condition $V^{(x)}(x \to d) = 0$ at the solid boundary; otherwise we would force the solid boundary to oscillate with a high frequency $\omega \gg \omega_{MRW}$. This implies that the density perturbations are confined to the strip $0 < x < x_s(T)$ which narrows with freezing, and the limit $x_s \ll d$ can be easily achieved. This effect results in a decrease of the spectrum of the EMP, since it is proportional to the Hall conductivity of the strip, $\sigma_{yx} \propto n_m(T)$.

Another important consequence of the stiff bulk area appearance is that a new EMP wave can propagate in the opposite (!) direction along the liquid–solid interface, since the interface can be considered as a rigid wall for typical EMP frequencies. The liquid density in the strip is not constant. Using the results of reference [3] we find that the spectrum of this mode can be written as

$$\omega \approx 2k_y\sigma_{yx}^{(0)}\frac{n_m(T)}{n_0}F(d/H) \tag{11}$$

where $F(d/H)$ is a function of order one [3]. The observation of waves propagating in the opposite direction can be an additional proof of the solidification transition in the 2D electron system.

It should be noted that both EMP and BDW propagation along the liquid strip should crucially depend on the viscosity of the electron liquid, since the boundary condition $V^{(y)}(x \rightarrow d) = 0$ in this case may induce a strong damping of the edge waves. In the same way the appearance of the liquid-crystal-like phase [14] would also affect these waves.

The quantum melting regime. In semiconductors the 2D electron system, due to its higher electron density, is primarily in the quantum melting regime: the Fermi energy $E_F = \pi\hbar^2 n_0/m$ is much larger than the thermal energy, and the phase state of the system is determined by the ratio $e^2\sqrt{\pi n_0}/(\epsilon E_F) \propto 1/\sqrt{n_0}$, where ϵ is the dielectric constant. This means that the solidification of the system caused by a strong magnetic field should start from the edge where the electron density is smaller. Therefore in the quantum regime the situation is inverse to that shown in figure 1. In this case we might have a 2D quantum electron liquid surrounded by a thin solid strip. The appearance of this solid strip should drastically affect the BDW mode due to the effective boundary condition $V^{(x)}(x \rightarrow 0) = 0$ at the liquid–solid interface. We could expect the BDW to vanish in this case. At the same time the rigid 'wall' of the solid strip imposes a sharp liquid density profile at the edge. According to [3], the influence of the finite width of the edge d on the EMP spectrum is essential. In the long-wavelength limit $|k_y|d \ll 1$,

$$\omega = \frac{2k_y\sigma_{yx}}{\epsilon}\left[\ln\frac{2}{|k_y|d} - C + C_1\right] \tag{12}$$

where $C = 0.577\ldots$ and C_1 depends on the shape of the density profile. The result for the sharp-density-profile approximation is obtained by making the following replacements in equation (12): $-C + C_1 \rightarrow 1$, $d \rightarrow d_B = 2\pi|\sigma_{xx}(\omega)|/(\epsilon\omega)$. In the strong-field limit the real density perturbation width d_B ($x_s < x \leqslant x_s + d_B$) becomes much smaller than d, which is not possible in the pure liquid state.

It should be emphasized that in real 2D electron systems it is very difficult to impose rigid boundary conditions at the edge, since the total force acting on an electron at the edge (the external force plus the force of the other electrons) should be zero. Therefore, in an experiment one excites the in-phase motion of the EMP and BDW [8]. The appearance of the solid strip at the edge might provide a unique possibility of suppressing one of the modes (BDW) and of studying properties of the pure EMP mode under the sharp-density-profile condition. This case is of the most interest if the bulk liquid state is close to an incompressible quantum Hall state.

5. Conclusion and summary

We have investigated the edge phenomena of a 2D Wigner solid in the presence of the magnetic field oriented normally. The analysis of the equations of motion of a 2D electron solid shows that no conventional EMP can exist in the pure solid state of the electron system. In the case of a movable boundary, new magneto-Rayleigh-wave solutions appear. The magnetic field dependence of the MRW spectrum is analogous to that of the EMP, but the MRW has a different dispersion and a much lower frequency.

In the case of the smooth electron density profile, the liquid–solid interface at the edge appears differently in the classical and quantum melting regimes, which will crucially affect the propagation of the EMP and BDW in the liquid phase of the electron system. These edge excitations can be used as an alternative probe of the Wigner solid.

Acknowledgments

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